

REALISTIC EXPANDING SOURCE MODEL FOR RELATIVISTIC HEAVY-ION COLLISIONS

Scott Chapman¹ and J. Rayford Nix¹

¹Theoretical Division
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

INTRODUCTION

An international search is currently underway for the quark-gluon plasma—a predicted new phase of nuclear matter where quarks roam almost freely throughout the medium instead of being confined to individual nucleons.^{1,2} Such a plasma could be formed through the compression and excitation that occur when nuclei collide at relativistic speeds. With increasing compression the nucleons overlap sufficiently that they should lose their individual identity and transform into deconfined quarks, and with increasing excitation the many pions that are produced overlap sufficiently that they should lose their individual identity and transform into deconfined quarks and anti-quarks.

Experimental identification of the quark-gluon plasma, as well as understanding other aspects of the process, will require knowing the overall spacetime evolution of the hot, dense hadronic matter that is produced in relativistic heavy-ion collisions. The spacetime evolution of this hadronic matter can in principle be extracted from experimental measurements of invariant one-particle multiplicity distributions and two-particle correlations in emitted pions, kaons, and other particles. The foundations for two-particle correlations were laid in the 1950s by Hanbury Brown and Twiss,³ who used two-photon correlations to measure the size of stars, and by Goldhaber et al.,⁴ who used two-pion correlations to measure the size of the interaction region in antiproton annihilation. Following this pioneering work, many researchers have already analyzed correlations among pions and among kaons produced in relativistic heavy-ion collisions in terms of simple models to obtain some limited information about the size and duration of the emitting source. However, because of the simplicity and/or lack of covariance of the models that have been used, the spatial and time extensions of the emitting source resulting from these analyses have frequently been intertwined, and most of the presently available results may therefore be regarded as exploratory.

SOURCE MODEL

We introduce here a new realistic expanding source model for invariant one-particle multiplicity distributions and two-particle correlations in nearly central relativistic heavy-ion collisions that contains nine adjustable parameters, which are necessary and sufficient to properly characterize the gross properties of the source during its freezeout from a hydrodynamical fluid into a collection of noninteracting, free-streaming hadrons. These nine physically relevant parameters fall into three categories of three parameters each, with the first category corresponding to the source's longitudinal motion, the second category corresponding to its transverse motion, and the third category corresponding to its intrinsic properties.

The three longitudinal parameters are the rapidity y_s of the source's center relative to the laboratory frame (in terms of which the velocity v_s of the source's center relative to the laboratory frame is given by $v_s = \tanh y_s$), the longitudinal spacetime rapidity η_0 of the right-hand end of the source in its own frame (in terms of which the velocity v_ℓ of the right-hand end of the source in its own frame is given by $v_\ell = \tanh \eta_0$), and the longitudinal freezeout proper time τ_f (in terms of which the longitudinal radius at the end of freezeout is given by $R_\ell = \tau_f \sinh \eta_0$). We assume that the source is boost invariant within the limited region between its two ends,^{5,6} and that it starts expanding from an infinitesimally thin disk at time $t = 0$.

The three transverse parameters are the transverse velocity v_t and transverse radius R_t of the source at the beginning of freezeout and a transverse freezeout coefficient α_t that is related to the width $\Delta\tau$ in proper time during which freezeout occurs and that determines the shape of the freezeout hypersurface. The transverse velocity at any point on the freezeout hypersurface is assumed to be linear in the transverse coordinate ρ . As illustrated in Fig. 1 for the reaction considered here,^{7–9} freezeout proceeds inward from the initial point $\rho = R_t$, $z = 0$ to the source's center and then to the source's ends.

Figure 1. Freezeout hypersurface, which specifies the positions in spacetime where the expanding hydrodynamical fluid is converted into a collection of noninteracting, free-streaming hadrons.

In our model, the coordinate-space freezeout surface is a hyperboloid of revolution of first one sheet and then of two sheets, which moves through the expanding source like the collapsing neck of a fissioning nucleus in the three-quadratic-surface shape parameterization.¹⁰

The three intrinsic parameters are the nuclear temperature T , the ratio μ_b/T of the baryon chemical potential to the temperature (from which μ_b itself can be readily calculated), and the fraction λ_π of pions that are produced incoherently. (The analogous quantity for kaons is held fixed at unity, in accordance with theoretical expectations.)

For a particular type of particle, the invariant one-particle multiplicity distribution and two-particle correlation are calculated in terms of a Wigner distribution function that includes both a direct term¹¹ and a term corresponding to 10 resonance decays,¹² namely the decay of meson resonances with masses below 900 MeV and strongly decaying baryon resonances with masses below 1410 MeV. Integration of the direct part of the Wigner distribution function over spacetime leads to the Cooper-Frye formula for the invariant one-particle multiplicity distribution.¹³

APPLICATION TO NEARLY CENTRAL Si + Au COLLISIONS

As an initial application, we apply our model to the analysis of invariant π^+ , π^- , K^+ , and K^- one-particle multiplicity distributions^{7,8} and π^+ and K^+ two-particle correlations⁹ for nearly central Si + Au collisions at $p_{\text{lab}}/A = 14.6$ GeV/ c . Figure 2

Figure 2. Comparison between model predictions and experimental data^{7,8} for the invariant π^+ one-particle multiplicity distribution. Results for successively increasing values of the particle rapidity y_p relative to the laboratory frame are multiplied by 10^{-1} for visual separation.

Figure 3. Dependence of the predicted K^+ two-particle correlation function C upon the longitudinal and “out” momentum differences, for fixed values of the other three quantities upon which C depends.

shows a comparison between model predictions and experimental data for an invariant one-particle multiplicity distribution, and Fig. 3 shows the dependence of a two-particle correlation function upon the longitudinal and “out” momentum differences.¹⁴

The nine adjustable parameters of our model are determined by minimizing χ^2 with a total of 1416 data points for the six types of data considered, so the number of degrees of freedom ν is 1407. The error for each point is calculated as the square root of the sum of the squares of its statistical error and its systematic error, with a systematic error of 15% for π^+ , π^- , and K^+ one-particle multiplicity distributions, 20% for the K^- one-particle multiplicity distribution, and zero for π^+ and K^+ two-particle correlations.^{7–9} The resulting value of χ^2 is 1484.6, which corresponds to an acceptable value of $\chi^2/\nu = 1.055$. The values of the nine parameters determined this way, along with their uncertainties at 99% confidence limits on all nine parameters considered jointly, are given in Table 1.

Table 1. Nine adjusted source freezeout parameters.

Parameter	Value and uncertainty at 99% confidence
Source rapidity y_s	1.355 ± 0.066
Longitudinal spacetime rapidity η_0	1.47 ± 0.13
Longitudinal freezeout proper time τ_f	$8.2 \pm 2.2 \text{ fm}/c$
Transverse freezeout velocity v_t	$0.683 \pm 0.048 c$
Transverse freezeout radius R_t	$8.0 \pm 1.7 \text{ fm}$
Transverse freezeout coefficient α_t	$-0.86^{+0.36}_{-0.14}$
Nuclear temperature T	$92.9 \pm 4.4 \text{ MeV}$
Baryon chemical potential μ_b/T	5.97 ± 0.56
Pion incoherence fraction λ_π	0.65 ± 0.11

Table 2. Additional calculated freezeout quantities.

Quantity	Value and uncertainty at 99% confidence
Source velocity v_s	$0.875^{+0.015}_{-0.016} c$
Longitudinal velocity v_ℓ	$0.900^{+0.023}_{-0.029} c$
Longitudinal freezeout radius R_ℓ	$16.9^{+5.6}_{-4.9} \text{ fm}$
Beginning freezeout time t_1	$3.1^{+2.5}_{-3.1} \text{ fm}/c$
Freezeout time t_2 at source center	$8.2 \pm 2.2 \text{ fm}/c$
Final freezeout time t_3	$18.8^{+5.8}_{-5.3} \text{ fm}/c$
Freezeout width $\Delta\tau$ in proper time ^a	$5.9^{+4.4}_{-2.6} \text{ fm}/c$
Baryon chemical potential μ_b	$554^{+34}_{-36} \text{ MeV}$
Strangeness chemical potential μ_s	$75^{+13}_{-12} \text{ MeV}$
Isospin chemical potential μ_i	$-5.3^{+1.0}_{-1.1} \text{ MeV}$
Number B_{proj} of baryons originating from projectile	$26.1^{+8.8}_{-6.6}$
Number B_{tar} of baryons originating from target	57^{+20}_{-15}
Total number B_{tot} of baryons in source	83^{+28}_{-21}
Baryon density n_1 at beginning of freezeout ^b	$0.057^{+\infty}_{-0.032} \text{ fm}^{-3}$
Baryon density n_s along symmetry axis	$0.0222^{+0.0097}_{-0.0069} \text{ fm}^{-3}$

^aCalculated under the additional assumption that the exterior matter at $z = 0$ that freezes out first has been moving with constant transverse velocity v_t from time $t = 0$ until time t_1 .

^bThe upper limit of ∞ for this quantity arises because the beginning freezeout time t_1 could be zero, at which time the shape is an infinitesimally thin disk.

From these underlying nine adjustable parameters we are able to calculate several additional freezeout quantities of physical interest and their uncertainties at 99% confidence limits, as shown in Table 2. For this reaction, the source-frame longitudinal velocity v_ℓ is $0.900^{+0.023}_{-0.029} c$ and the transverse freezeout velocity v_t is $0.683 \pm 0.048 c$. These relatively large values mean that the contribution to the energy of the produced pions and kaons from collective flow is substantial, which leads to a moderately low freezeout nuclear temperature T of $92.9 \pm 4.4 \text{ MeV}$. The transverse radius R_t at the beginning of freezeout is $8.0 \pm 1.7 \text{ fm}$, and the longitudinal radius R_ℓ at the end of freezeout is $16.9^{+5.6}_{-4.9} \text{ fm}$.

We also investigated the dependence of χ^2 upon nuclear temperature T in the interval 50 to 250 MeV, with the remaining eight parameters varied. On both sides of the absolute minimum at $T = 92.9 \text{ MeV}$, χ^2 rises steeply. However, for values of T greater than 129 MeV, the minimum solution switches to an unphysical branch corresponding to the nearly instantaneous freezeout of a thin disk of large radius.

The predictive power of the model can be ascertained by comparing calculated and experimental quantities that were not included in the adjustment. For example, the number B_{proj} of baryons in the source that originate from the projectile is calculated to be $26.1^{+8.8}_{-6.6}$, which agrees well with the value 28 corresponding to a Si projectile. In addition, although proton data were not used in our adjustment, the calculated invariant proton one-particle multiplicity distribution agrees moderately well with the experimental distribution for particle rapidities relative to the laboratory frame of 1.3 or greater. For lower rapidities, however, the data show far more low- p_t protons than are predicted. We regard these excess protons as target spectators that have interacted somewhat, but not enough to be considered part of the expanding system.

CONCLUSIONS

We have introduced a new realistic expanding source model containing the necessary and sufficient number of adjustable parameters for properly describing invariant one-particle multiplicity distributions and two-particle correlations in nearly central relativistic heavy-ion collisions. On the basis of this model, we have extracted the freezeout properties of the the hot, dense hadronic matter that is produced in nearly central Si + Au collisions at $p_{\text{lab}}/A = 14.6$ GeV by minimizing χ^2 with a total of 1416 data points for invariant π^+ , π^- , K^+ , and K^- one-particle multiplicity distributions and π^+ and K^+ two-particle correlations. The extracted properties include not only the nine adjustable parameters contained in the model but also several additional freezeout quantities that can be calculated in terms of the underlying nine parameters. In all cases, we have presented uncertainties at 99% confidence limits.

For the reaction considered here, the extracted freezeout nuclear temperature T is 92.9 ± 4.4 MeV, the local-rest-frame baryon density n_s along the symmetry axis is $0.0222^{+0.0097}_{-0.0069}$ fm $^{-3}$, and the longitudinal freezeout proper time τ_f is 8.2 ± 2.2 fm/ c . We hope that our model will be used in the future to systematically analyze other existing and forthcoming data on invariant one-particle multiplicity distributions and two-particle correlations to extract freezeout properties as functions of bombarding energy and target-projectile combinations. A sharp discontinuity in the dependence of the extracted freezeout properties upon these quantities could signal the onset of the long-sought-for quark-gluon plasma or other new physics.

ACKNOWLEDGMENTS

We are grateful to Arnold J. Sierk for his participation in the early stages of this work and to T. Vincent A. Cianciolo for permitting us to use his preliminary data on two-particle correlations in our adjustments. This work was supported by the U. S. Department of Energy.

REFERENCES

1. H. Satz, *Ann. Rev. Nucl. Part. Sci.* 35:245 (1985).
2. “Quark Matter ’95, Proc. Eleventh Int. Conf. on Ultra-Relativistic Nucleus-Nucleus Collisions, Monterey, California, 1995,” *Nucl. Phys.* A590:1c (1995).
3. R. Hanbury Brown and R. Q. Twiss, *Phil. Mag.* 45:663 (1954).
4. G. Goldhaber, W. B. Fowler, S. Goldhaber, T. F. Hoang, T. E. Kalogeropoulos, and W. M. Powell, *Phys. Rev. Lett.* 3:181 (1959).
5. F. Cooper, G. Frye, and E. Schonberg, *Phys. Rev. D* 11:192 (1975).
6. J. D. Bjorken, *Phys. Rev. D* 27:140 (1983).
7. T. Abbott et al. (E-802 Collaboration), *Phys. Rev. C* 50:1024 (1994).
8. National Nuclear Data Center, WWW URL <http://necs01.dne.bnl.gov/html/nndc.html>, NNDCC Online Data Service, Data Base CSISRS, Accession Number C0501.
9. T. V. A. Cianciolo, private communication (1996).
10. J. R. Nix, *Nucl. Phys.* A130:241 (1969).
11. J. Bolz, U. Ornik, M. Plümer, B. R. Schlei, and R. M. Weiner, *Phys. Lett.* B300:404 (1993).
12. L. Montanet et al. (Particle Data Group), *Phys. Rev. D* 50:1173 (1994).
13. F. Cooper and G. Frye, *Phys. Rev. D* 10:186 (1974).
14. G. Bertsch, M. Gong, and M. Tohyama, *Phys. Rev. C* 37:1896 (1988).

Si + Au, central 7%
 $p_{\text{lab}}/A = 14.6 \text{ GeV}/c$

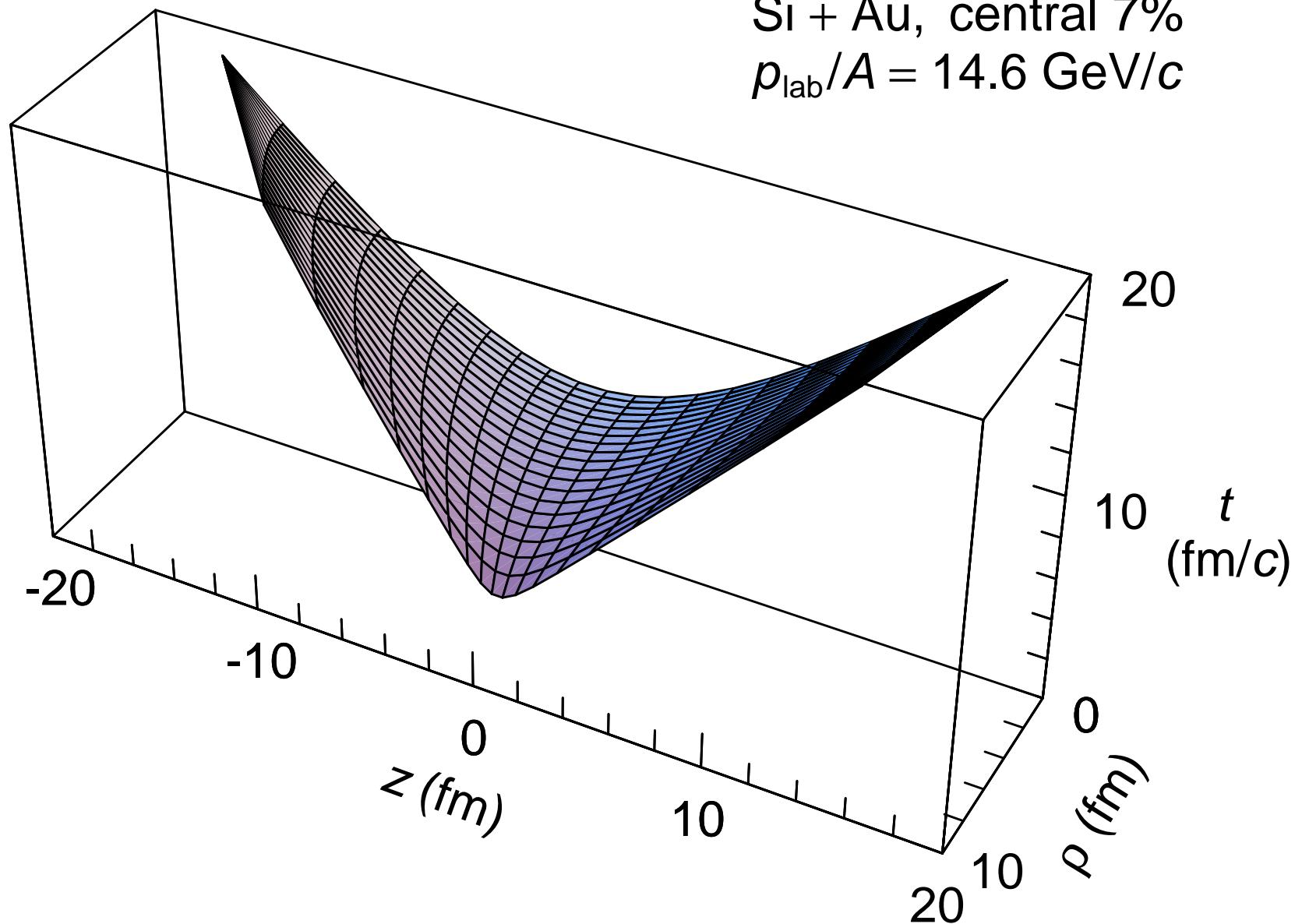


Figure 1

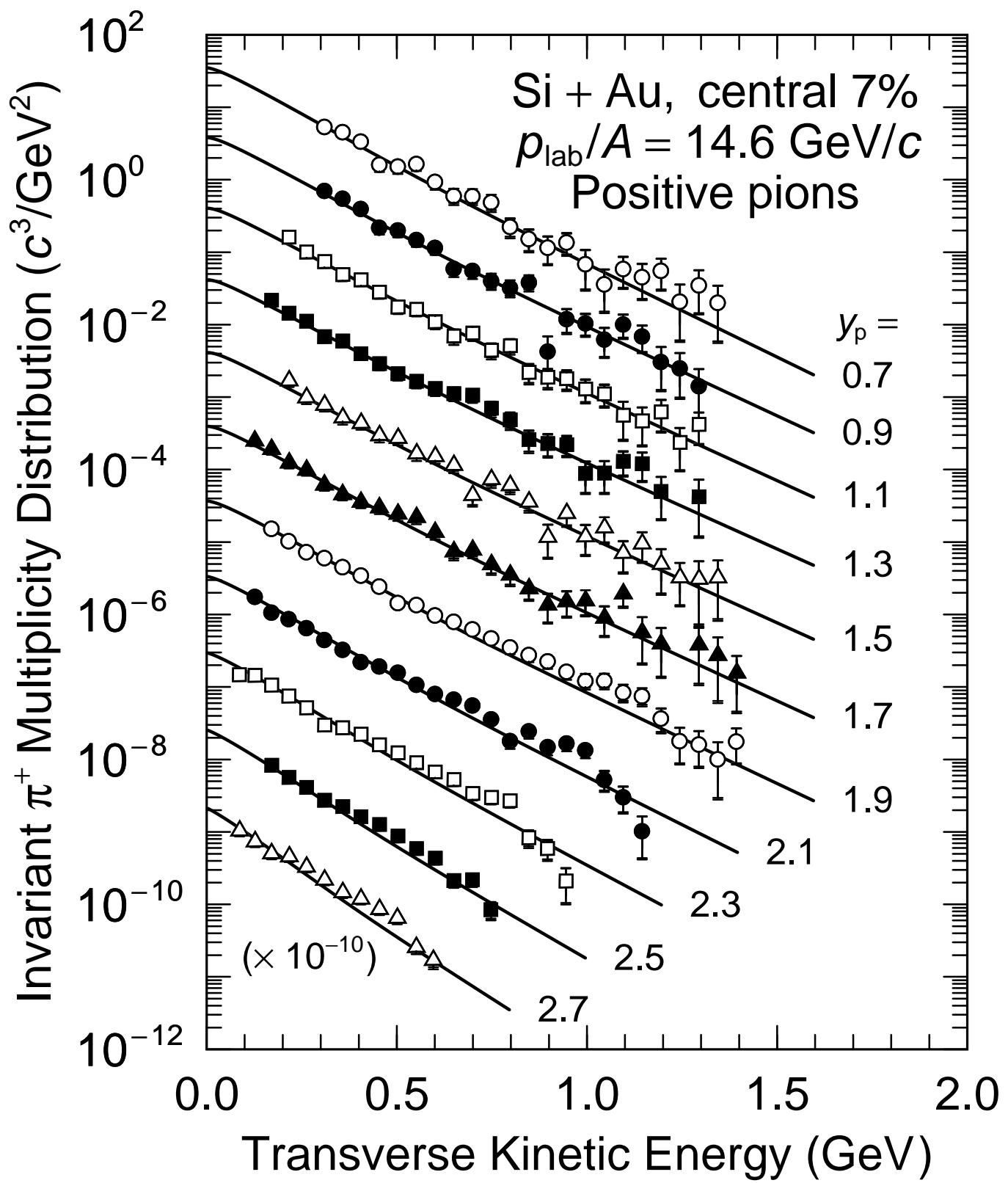


Figure 2

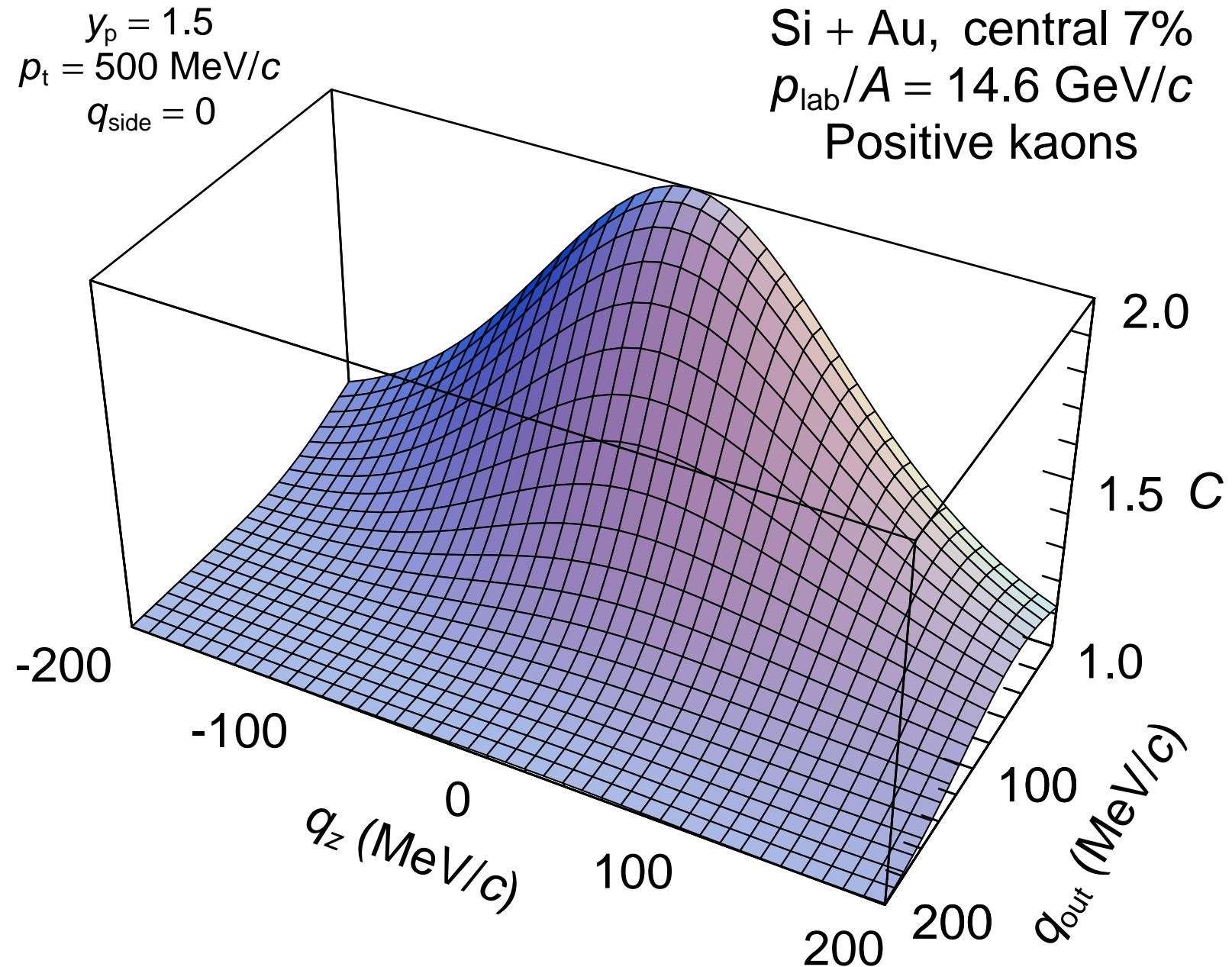


Figure 3